芸術科学会論文誌 Vol.11, No.4, pp. 118 – 128

# 炭酸水から生じる気泡のビジュアルシミュレーション

金森 由博 1)(正会員)　西川 武志 2)
Yonghao Yue2)(正会員)　西田 友是 2)(正会員)
1) 筑波大学　2) 東京大学

# Visual Simulation of Bubbles in Carbonated Water[1]

**Yoshihiro Kanamori**1)　**Takeshi Nishikawa**2)
**Yonghao Yue**2)　**Tomoyuki Nishita**2)

1) University of Tsukuba　2) The University of Tokyo
kanamori@cs.tsukuba.ac.jp, {takeshi,yonghao,nis}@nis-lab.is.s.u-tokyo.ac.jp

**概要**
本論文では，炭酸水における気泡の生成，成長および運動の簡易なシミュレーション手法を提案する．気泡は，グラス，マドラー，炭酸水中の微小なほこりといった物体表面上の，小さな空気孔から発生することが知られている．気泡の数や生成速度は，物体の材質によって異なる．さらに物体の材質によって物体表面での気泡の挙動も異なる．プラスチックやアクリルといった材質の場合，気泡はゆっくりと表面を滑るように浮上するが，ステンレスのような材質ではそのような挙動は見られない．提案手法ではこのような差異を，空気孔の数を制御し，気泡に対する物体表面への付着力を導入することによって表現できる．炭酸水中の気泡は十分に小さいので，提案法では気泡を変形しない球体として扱う．格子法を用いて炭酸水の流れを計算することで提案法では気泡と炭酸水の対流を同期させている．さらに，複雑な気泡の挙動を再現するため気泡同士の衝突や融合も考慮している．

**Abstract**
In this paper, we propose a simple method for simulating the generation, growth and motion of bubbles in carbonated water. Bubbles are known to be generated at small air pockets on the surfaces of objects, such as a glass or a stirrer, and on the microscopic dusts in carbonated water. The number of bubbles and their growth rates differ according to the materials of the objects. In addition, the motion of bubbles also differs according to the materials; bubbles slowly glide on plastic and acrylic surfaces whereas such motions cannot be seen on a stainless-steel surface. Our method can represent such differences by controlling the number of generation points (i.e., air pockets) and applying adhesion forces to make bubbles stick on the surfaces. Because bubbles in carbonated water are sufficiently small, our method regards bubbles as non-deformable spherical particles. We simulate the flow in carbonated water using a grid-based method, together with the two-way coupling technique between bubbles and the convective flow. Additionally, the collisions and fusions among bubbles are also handled to reproduce the complex behavior of bubbles.

[1] この論文は NICOGRAPH International 2012 に投稿した論文 [1] を芸術科学会論文誌に投稿するものである．

# 1 Introduction

Water is so ubiquitous in our daily life that we unconsciously memorize its fascinating behavior. The demands of creating realistic water animations have therefore been quite high in computer graphics. Until recently, water animations have been somewhat artificial in spite of the progress of simulation techniques, because bubbles have been ignored to reduce the computational cost.

Recently, bubble simulations have gained attentions to reproduce various phenomena including boiling [3] and air trapping [5]. Cleary *et al*. [6] handled simulations of bubbles in carbonated water; however, their method suffers from the quite high computation cost, and cannot sufficiently reproduce the interesting behavior of bubbles around object surfaces, which is essential to represent the realistic appearance of carbonated water.

Here, we describe the behavior of bubbles in carbonated water. Bubbles are known to be generated at small air pockets on the surfaces of objects, such as a glass or a stirrer, and on the microscopic dusts in carbonated water. In this paper, such air pockets are called *generation points*. The number and capacities of generation points largely depend on the materials of objects, resulting in the differences of the amount and sizes of generated bubbles. For example, bubbles are rapidly generated from a wooden stirrer due to the vast amount of generation points on its porous surface (Figure 1). The duration in which each bubble adheres to the surface also differs according to materials. Such difference of the duration yields various sizes of bubbles because bubbles keep growing by absorbing over-condensed gases resolved in carbonated water. After leaving the surface, bubbles rise along the convection flow caused by the stream of bubbles.

In this paper, we propose a method for generating realistic animations of bubbles in carbonated water, with special interests in the behavior of bubbles around object surfaces, as described above. Our method controls the amount of bubbles by adjusting the number of generation points on the object surface. Regarding the phenomenon that bubbles adhere to object surfaces, the physical principle is not clearly known to the best of our knowledge. Instead of seeking the faithful physical law, we simply model the adhesion by introducing a friction-like force whose magnitude is proportional to that of the normal force from the surface. Consequently, our method can represent not only the different durations of bubble adhesion but also an interesting phenomenon that bubbles climb along smooth surfaces (Figure 1, right). Our method handles the bubbles' motion due to the convection flow by representing bubbles with particles and the flow with the Eulerian grid-based method [9], and computing the two-way coupling [10] between the bubbles and the flow. The reality of the bubble motion is further enhanced by handling the rebound and fusion of bubbles when they collide. Unlike Cleary *et al*.'s model, our simple model is computationally inexpensive, and thus suited for interactive applications.

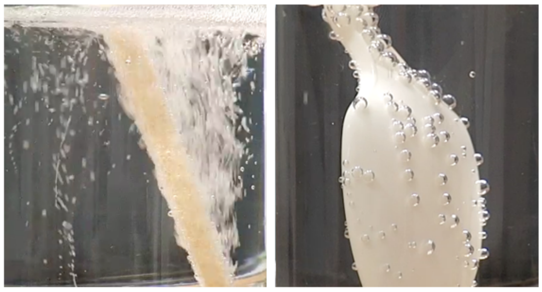

Figure 1: Photographs of objects in carbonated water. Left: a wooden stirrer, right: a plastic spoon. The amount of bubbles differs due to the difference of materials.

# 2 Related Work

Simulations of liquids including water belong to the most important themes in the research of computer graphics. Here we elaborate on the related methods that handle the behavior of bubbles in liquids.

Hong *et al*. [8] simulated deformations and coalescence of large bubbles by combining the *Volume of Fluid* (VOF) *method* and the *front-tracking method*. Song *et al*. [11] used the *Constrained Interpolation Profile* (CIP) method to accurately calculate advection in the simulation of multiphase flows. They handled bubbles that are generated by the air trapped when objects are fallen into the water. There are also several methods for bubbles generated via boiling of liquids [2, 3, 12]. All of these methods represent bubbles with grids; in the case of small bubbles in carbonated water, the methods require quite fine grids, and thus become computationally expensive.

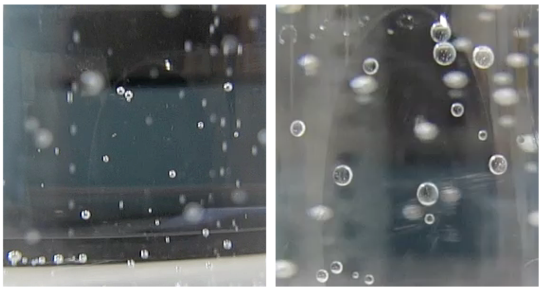

Figure 2: Photographs of the surfaces of containers containing carbonated water. Left: a glass, right: a plastic bottle, taken at the same distance. The sizes of bubbles also differ due to the difference of materials.

Müller *et al*. [13] used the *Smoothed Particle Hydrodynamics* (SPH) method to simulate the interaction between multiple fluids and state transitions including boiling. Their method can handle the deformation and coalescence of bubbles by representing both the air and liquids with particles. Greenwood *et al*. [5] dealt with the trapped air as bubbles by approximating bubbles in liquids as spheres and simulating the generation of bubbles using the particle level set method. Hong *et al*. introduced the two-way coupling of a grid-based method and the SPH method, and represented the interaction between rising bubbles and the water flow. Thürey *et al*. [14] accomplished simulations of the interaction between bubbles and liquids represented with a height field in real time. These previous methods do not target bubbles in carbonated water, and thus do not take into account the mechanism of their generation. Recently, Kim *et al*.[4] introduced a pratical simulation method for dispersed bubble flow. Although their method does not target carbonated water as well as bubble collisions or merging, combining it with our method will improve the motion of rising bubbles.

Cleary *et al*. [6] simulated the generation of bubbles in carbonated water using the SPH method. In their model, the resolved gas in carbonated water is propagated by water particles via a diffusion process, and is provided to the nucleation sites. Although their method can handle highly dynamic animations as shown in their example of Ale poured into a glass, the use of a large number of particles makes their method quite expensive, as they reported that a simulation takes more than ten days. Furthermore, their method neither handles the adhesion of bubbles onto object surfaces nor the differences of sizes and generation rates of bubbles according to the materials, as shown in Figures 1 and 2.

Our method can handle such differences by introducing adhesion forces that make bubbles stick on object surfaces. Unlike Cleary *et al*.'s method, our method represents bubbles with particles and the water with a coarse grid together with the two-way coupling [10] of the two. Our method is much simpler and less expensive than theirs.

# 3 Bubble Simulation

## 3.1 Overview

Our method represents bubbles in carbonated water with non-deformable spheres, because each bubble in carbonated water is roughly one millimeter in diameter and can be regarded as such [15]. Bubbles that adhere at generation points are also represented with non-deformable spheres.

We target the behavior of bubbles inside carbonated water and do not handle bubbles floating on the water surface, which we would like to deal with in a way similar to Cleary *et al*. [6] in future work. Our method controls the amount of generated bubbles by adjusting the number of generation points. Bubbles stay at their generation points and keep growing when the external forces that attempt to move bubbles are weak, and then start to move when the external forces become sufficiently strong. We reproduce this phenomenon by introducing *adhesion forces* based on the analogy of friction forces. The adhesion forces are calculated so that their directions are opposite to the motion of bubbles and their magnitudes are linearly proportional to those of the normal forces from object surfaces. The coefficients of adhesion forces affect the sizes of bubbles when they are released from the surface; larger coefficients help bubbles grow larger, while smaller coefficients let them leave earlier. Note that we assume that carbon dioxide ($CO_2$) is sufficiently resolved in the water for the growth of bubbles.

Our method computes the water flow using the grid-based method by Stam [9] and handles the interaction between bubbles and the flow. Specifically, we use the two-way coupling technique [10] to represent the convection flow caused by drifting bubbles as well as the motions of bubbles influenced by the flow. Additionally, when bubbles collide, our method lets them repel or merge with used-specified probability.

## 3.2 The Behavior of Bubbles on Object Surfaces

Our method randomly places generation points on the object surfaces with the user-specified density according to the materials. For example, a porous object like a wooden stirrer (Figure 2, left) are assigned with a large number of generation points, while a plastic object (Figure 2, right) not so many.

The duration in which each bubble sticks to the surface varies according to the material of the surface, resulting in different sizes and growth rates of bubbles. To represent this phenomenon, we introduce adhesion forces that let bubbles stick to the surface. To the best of our knowledge, the theory of the adhesion is not clearly known. Here we introduce a simple model based on the consideration of external forces against bubbles; a bubble at a generation point is pushed onto the object surface by hydraulic pressure. According to Newton's third law of motion, the bubble receives a normal force in the opposite direction of the pressure. The bubble also receives a buoyancy force from the surrounding water, and possibly a collision force as well. To maintain the equilibrium against these external forces, we assume that the bubble receives an *adhesion force* similar to a friction force, and its magnitude is linearly proportional to that of the normal force. Specifically, in our model, the bubble is assumed to receive what we call a *static adhesion force* $\mathbf{F}_{st}$ in the stationary state;

$$\mathbf{F}_{st} = -\mu|\mathbf{N}|\frac{\mathbf{g}}{|\mathbf{g}|}, \tag{1}$$

where $\mu$ is a coefficient for static adhesion, $\mathbf{N}$ is the normal force (illustrated as the green arrow in Figure 3) and $\mathbf{g}$ is the gravity. To decide whether the bubble stays at the generation point or leaves there, we compare the magnitudes of the static adhesion force and the total external forces in each time step of the simulation. Larger static adhesion forces let bubbles stay longer at the generation point and thus grow larger by absorbing the resolved gas, while smaller forces let them leave earlier even when they are relatively small. After leaving the generation point, the bubble may immediately take off for the water surface from the object surface or glide up along the object surface. Such behavior depends on the material of the object. For example, on plastic and acrylic surfaces bubbles slowly glide, while on a stainless-steel surface they do not. To represent such a phenomenon, we introduce a *dynamic adhesion force*, similar to a dynamic friction force. Its magnitude is also linearly proportional to that of the normal force;

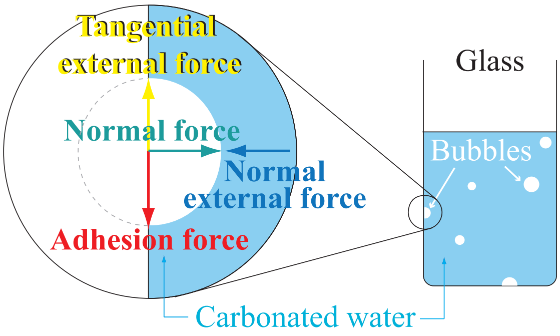


Figure 3: Relationship between adhesion and external force. The blue arrow represents the total of force from hydraulic pressure. The green, red and yellow arrows represent the normal force, the adhesion force and the horizontal component of the external force, respectively.

$$\mathbf{F}_{dy} = -\mu'|\mathbf{N}|\frac{\mathbf{g}}{|\mathbf{g}|}, \tag{2}$$

where $\mu'$ is a coefficient for dynamic adhesion. $\mu'$ is set small for plastic or acrylic surfaces and large for stainless-steel surfaces.

## 3.3 Grid-based Fluid Simulation

Our method simulates the water flow using a grid-based method. The simulation space is subdivided into a coarse grid, and the velocity of the fluid is assigned for each grid cell. The velocities are updated based on the Navier-Stokes equations in each time step;

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} + \nu\nabla^2\mathbf{u} - \frac{1}{\rho}\nabla p + \mathbf{f}, \tag{3}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{4}$$

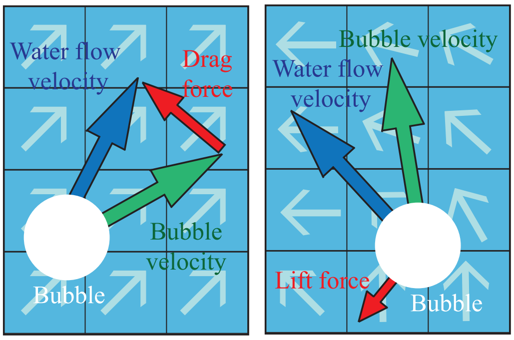


Figure 4: Illustrations of the drag force, the lift force and the interactions between bubbles and the water flow from left to right.

where $\mathbf{u}$ is the velocity, $p$ is the pressure, $\rho$ is the fluid density and $\mathbf{f}$ is the external forces including the gravity. We solve these equations using *Stable Fluids* [9].

## 3.4 Interaction between Bubbles and Water Flow

We compute the interaction between bubbles and the water flow using the two-way coupling technique [10]. See Figure 4 for illustrations. The flow provides *drag* and *lift* forces to bubbles. The drag forces $\mathbf{F}_d$ work so that bubbles move along the flow. The lift forces $\mathbf{F}_l$ let bubbles follow vortices in the flow.

$$\mathbf{F}_d = -k_{drag}\frac{m_i}{r_i}|\mathbf{v}_i - \mathbf{u}_i|(\mathbf{v}_i - \mathbf{u}_i), \quad (5)$$

$$\mathbf{F}_l = -k_{lift} m_i(\mathbf{v}_i - \mathbf{u}_i) \times \omega_i, \quad (6)$$

where $m_i$ and $r_i$ are the mass and the radius of the bubble particle $i$, $\mathbf{u}_i$ is the water velocity at the position of bubble $i$, which is linearly interpolated from the velocities at surrounding grid points. $k_{drag}$ is the coefficient for the drag force and $k_{lift}$ is the coefficient for the lift force. $\mathbf{v}_i$ is the velocity of bubble particle $i$ and $\omega_i$ is the vorticity at the position of bubble particle $i$. Conversely, the flow is affected by the bubbles as well; the total force among bubbles near each grid cell is calculated, and then its inverse-directional force is added as an external force in Equation (3).

## 3.5 Interaction among Bubbles

We can observe repulsion and coalescence of bubbles when they collide, especially when they are gliding on an object surface. Our method detects collisions of bubbles, and then stochastically determines whether they repel or coalesce into one. The probability is specified by the user.

When bubbles are merged, our method erases collided bubbles and creates a new bubble whose volume is the total of those of collided bubbles and whose position is the centroid of them. The radius of the new bubble is calculated according to its volume.

When bubble particles $i$ and $j$ repel, the following repulsion force $\mathbf{F}_c$ is added to bubble particle $i$, and also added to bubble particle $j$ in the opposite direction;

$$\mathbf{F}_c = m_i(k_{col}\Delta\mathbf{x} + k_{damp}\mathbf{v}_n), \quad (7)$$

$$\Delta\mathbf{x} = \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|}((r_i + r_j) - |\mathbf{x}_i - \mathbf{x}_j|), \quad (8)$$

where $k_{col}$ is the stiffness coefficient, $k_{damp}$ is the damping coefficient and $\mathbf{v}_n$ is the normal component of the relative velocity.

## 3.6 The Motion Equation for Bubbles

In this section, we summarize the external forces applied to bubbles and the motion equation for the bubbles.

A bubble receives a buoyancy force $\mathbf{F}_b$ due to the difference of densities against water;

$$\mathbf{F}_b = (m_i - \rho_{liq}V_{bub})\mathbf{g}, \quad (9)$$

where $\rho_{liq}$ is the density of carbonated water, $V_{bub}$ is the volume of a bubble. While a bubble is inside carbonated water, it keeps growing by absorbing the resolved gas in the amount proportional to its surface area.

In summary, the total of the external forces $\mathbf{F}_{all}$ for a bubble is the summation of the buoyancy force (Equation (9)), the repulsive force from collision (Equation (7)), the drag and the lift forces from the water flow (Equations (5) and (6));

$$\mathbf{F}_{all} = \mathbf{F}_b + \mathbf{F}_c + \mathbf{F}_d + \mathbf{F}_l. \quad (10)$$

In addition, our method adds the dynamic adhesion forces to gliding bubbles (Section 3.2). Our method then updates the velocity and position of the bubble according to the law of motion using the total forces;

$$\frac{d\mathbf{v}_i}{dt} = \frac{\mathbf{F}_{all}}{m_i}, \tag{11}$$

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \tag{12}$$

where $m_i$ is the mass of bubble particle $i$.

# 4 Results

This section shows the results of 2D and 3D simulations using our method. The implementation was written in C++ using OpenGL. All experiments in this paper were conducted on a PC with an Intel Core i7 extreme 975 3.33GHz processor and 3GB RAM. As for rendering, the 2D results are displayed by representing bubbles with texture-mapped quads. The 3D results are rendered using an off-line renderer *POV-Ray*.

Figure 5 shows gliding motions of bubbles on surfaces with different materials. The motion of each bubble differs due to the differences in the coefficient of dynamic adhesion (Section 3.2). In this simulation results, the bubble on the surface that has a lower coefficient of dynamic adhesion moves faster.

Figure 6 shows the 2D simulation results of the interaction between bubbles and the water flow, assuming that a wooden stirrer is placed obliquely in the tilted rectangle area. The top and middle rows of Figure 6 show the results with and without interactions between bubbles and the water flow, respectively. The difference of bubbles' motion can be observed as shown in the areas with red circles. Figure 7 shows the results with different amounts and sizes of bubbles. The top result of Figure 7 contains 5,000 generation points while the bottom one 500. The computational times for each time step (without rendering) are 1.6 milliseconds for the top and 1.1 milliseconds for the bottom, respectively. The coefficients for static adhesion (Section 3.2) are 0.001 for the top result and 0.04 for the bottom result, respectively. The smaller coefficient in the top yields smaller bubbles and the faster growth rate than those in the bottom. Figure 8 shows a comparison between a photograph and the result combining 2D simulation and a background image. The corresponding accompanying video demonstrates that visually plausible animations can be obtained using our method. There are 1,000 generation points on the grape image area. The coefficient of static adhesion force is 0.06.

Figure 9 shows results of 3D simulations, captured every 30 time steps. There are a cylindrical glass and a blue glass ball. The ball has 500 generations points on its surface. We assume that there are no generation points on the wall of the cylindrical glass. Bubbles stay at their generation points for a while due to the static adhesion forces. Then, bubbles glide up along the surface. Regarding the attached bubbles, we let them slightly intrude into the object surface to represent their adhesion. To show the glide motions more clearly, we show in Figure 10 a near-view of another 3D simulation with the generation points reduced to 50 and other parameters remained the same as Figure 9. When the total external forces become larger than the static adhesion forces, bubbles leave their generation points and then glide up along the object surface. The computational time for each time step (without rendering) is 79 milliseconds.

Note that we cannot compare our results with those of previous methods in the same condition because, to the best of our knowledge, no previous methods focused on the generation and growth of bubbles on a surface. In terms of computational time, our method is much more suited to interactive applications than Clearly et al.'s method. Regarding parameter tuning, the coefficient for static adhesion force and the number of generation points are important to represent the differences of surface materials, as demonstrated in Figure 7.

# 5 Conclusions and Future Work

This paper has proposed a simple method for simulating bubbles in carbonated water, focusing on their behavior around object surfaces. Our method controlled the amount of bubbles generated from object surfaces by adjusting the number of generation points. To represent various sizes and on-surface gliding motions of bubbles, our method introduced *static* and *dynamic adhesion forces*, which make the bubbles stick on the surfaces. Our method also handled collisions and fusions among bubbles, and the two-way coupling between bubbles and the flow in carbonated water.

For future work, we would like to verify the physical validity of our method and represent more realistic behaviors of bubbles, including floating bubbles on the surface of liquids.

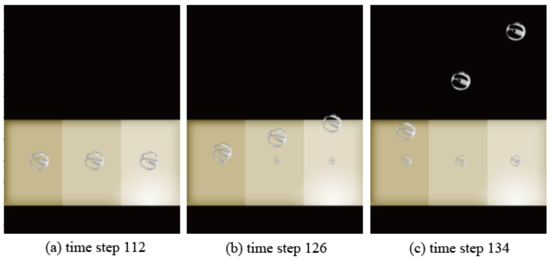


Figure 5: Sequence of different motions of bubbles on the surfaces. In each image, brown partitions are surfaces and the coefficient of dynamic adhesion is different in each partition (1.0, 0.6, and 0.3 from left to right, respectively). Bubbles on the more right side rise faster because bubbles receive less dynamic adhesion forces which are proportional to the coefficient of dynamic adhesion.

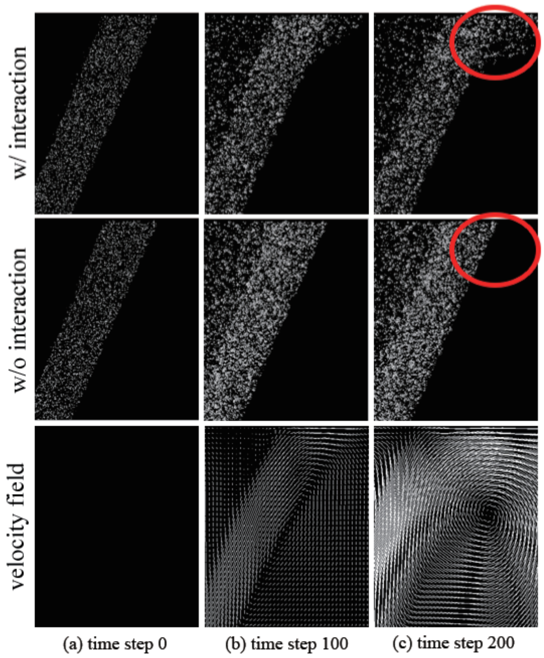


Figure 6: Top: 2D simulation results with interactions between bubbles and carbonated water flow, middle: 2D simulation results without interactions between bubbles and carbonated water flow, bottom: The velocity field of carbonated water flow.

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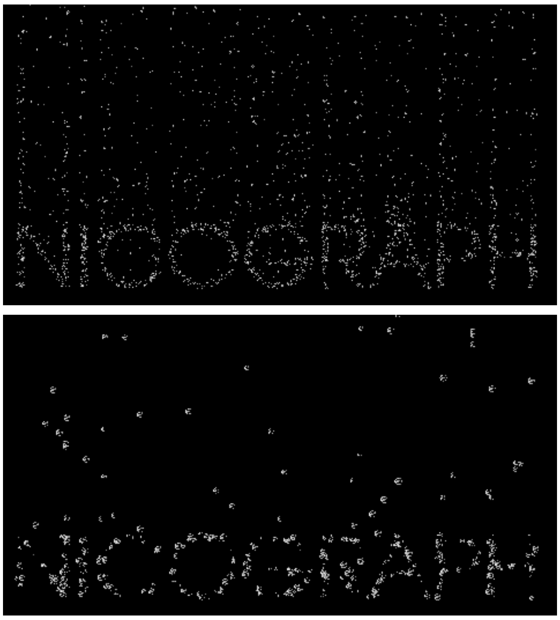


Figure 7: The amount of bubbles differ according to different materials of the object. The size of bubbles differ according to the coefficients of static adhesion force. The top and bottom simulation results have 5,000 generation points with $\mu$ = 0.001 and 500 generation points with $\mu$ = 0.04, respectively.


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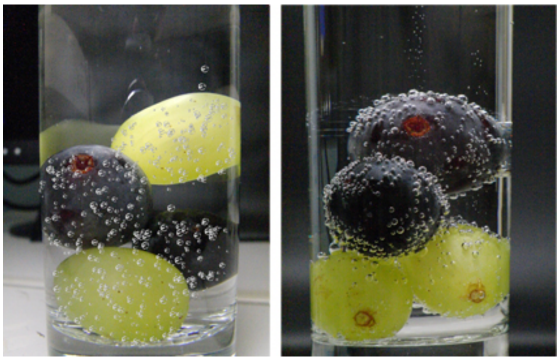
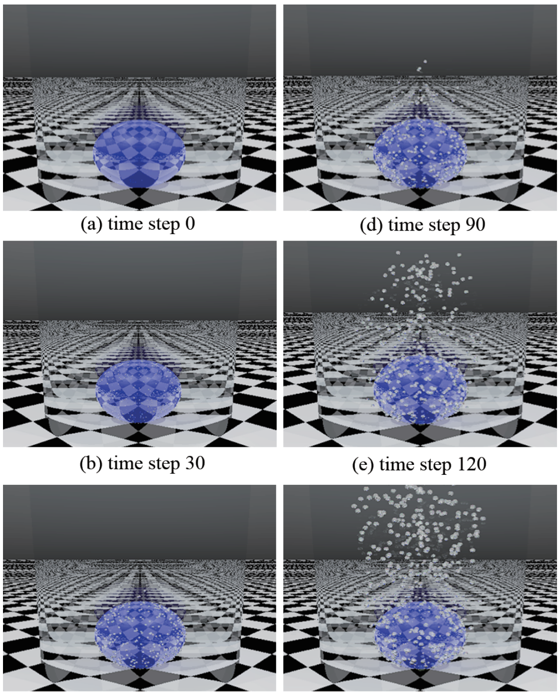
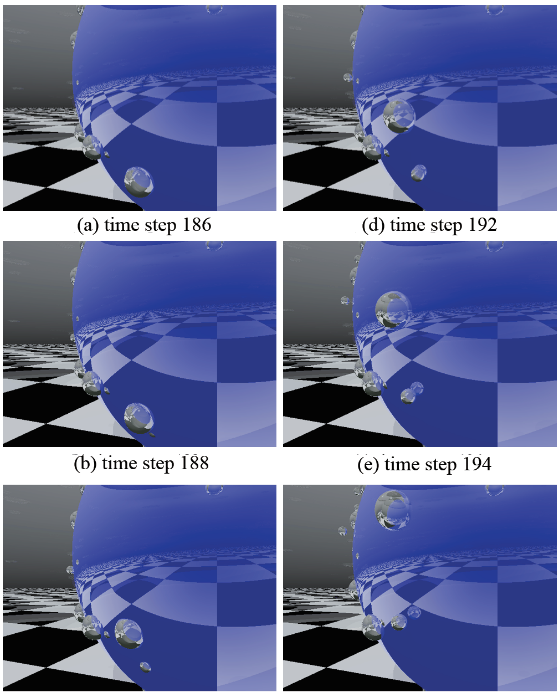


Figure 8: Comparison of a 2D simulation result and a photograph. Left: a simulation result, right: a photograph.



(a) time step 0 (d) time step 90
(b) time step 30 (e) time step 120
(c) time step 60 (f) time step 150

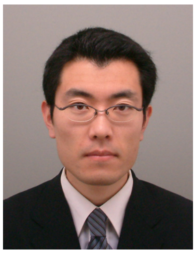
Figure 9: 3D simulation of bubbles rising in a cylindrical cup.



(a) time step 186 (d) time step 192
(b) time step 188 (e) time step 194
(c) time step 190 (f) time step 196

Figure 10: Bubbles glide on the surface of the spherical object.

**Yoshihiro Kanamori**

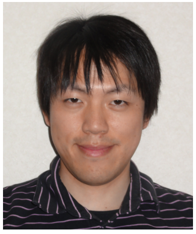


Yoshihiro Kanamori received his B.S., M.S., and Ph.D. degrees in computer science from the University of Tokyo, Japan, in 2003, 2005, and 2009, respectively. Since April 2009, he has been an assistant professor at the University of Tsukuba. His research interests center in computer graphics including rendering and visual simulations. He now studies image edting techniques for reproducing real-world phenomena. He is a member of IIEEJ and IPSJ.

**Takeshi Nishikawa**

Takeshi Nishikawa received his M.S. in Science from the University of Tokyo, Japan in 2010. His research in the University of Tokyo focused on computer graphics. He now works for SOFTBANK CORP.

**Yonghao Yue**



Yonghao Yue is an assistant professor at the Department of Complexity Science and Engineering, Graduate School of Frontier Sciences, the University of Tokyo, Japan, since 2011. He is also an assistant professor at the Department of Information Science, School of Science, the University of Tokyo, Japan, since 2012. His research interests center in computer graphics, including realistic image synthesis, global illumination, Monte Carlo techniques and physically-based simulations. He received his B.S., M.S. and Ph.D. degrees in Computer Science from the University of Tokyo, Japan, in 2005, 2007 and 2011, respectively. He is a member of ACM, IIEEJ, IPSJ and ITE.

**Tomoyuki Nishita**

Tomoyuki Nishita is a professor in the Department of Complexity Science and Engineering (also in the Department of Information Science) at the University of Tokyo, Japan since 1998. He received his BE, ME and Ph.D in Engineering in 1971, 1973, and 1985, respectively, from Hiroshima University. He taught at Fukuyama University from 1979 to 1998 He was an associate researcher in the Engineering Computer Graphics Laboratory at Brigham Young University from 1988 to 1989. His research interests center in CG including lighting/shading models (radiosity), natural phenomena, real-time rendering, geometric modeling, and non-photorealistic rendering. He is one of the pioneers of radiosity method. Dr. Nishita received Research Award on Computer Graphics from Information Processing Society of Japan in 1987, and also received Steaven A. Coons award from ACM SIGGRAPH in 2005. He was a president of The Institute of Image Electronics Engineers of Japan in 2009-2010.