Modeling of Volcanic Clouds using CML*

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In this paper, the model of volcanic clouds for computer graphics using the Coupled Map Lattice (CML) method is proposed. In this model, the Navier-Stokes equations are used, and the equations are solved by using the CML method that can be applied as an efficient fluid solver. Moreover, to generate desired shape of volcanic clouds, some parameters that allow intuitively control of the shape are provided. Hence, in this system, the behavior of the volcanic clouds can be calculated in practical calculation time, and by only changing some parameters various shapes of the volcanic clouds can be generated. Therefore, photo-realistic images/animations of various shapes of volcanic clouds can be created efficiently by the proposed approach.

Keywords: volcanic clouds, coupled map lattice, modeling, visualization, animation, computational fluid dynamics, cellular automaton

1. INTRODUCTION

The modeling of volcanic clouds is useful for natural disaster simulations, entertainment (e.g. games, movies), etc. However, there is not much research on the modeling of volcanic clouds. Especially, in the field of computer graphics, this kind of research has almost not been done so far. Although there are several commercial modeling products, for example [33], that can generate volcanic cloud images, they can only obtain the motion of volcanic clouds according to the orbits of some particles that are set by professional users. Therefore, we propose a modeling method for volcanic clouds using the Coupled Map Lattice (CML) method. The CML method [15, 16, 27-31] is one kind of cell dynamics, which generates patterns. The advantages of using the CML method are: (1) it is easy to implement and (2) it has low computational cost. Moreover, to generate desired shape of volcanic clouds, some parameters that allow intuitive control of the shape are provided, as is a graphical user interface (GUI) for interactively setting of the parameters. The goals of this approach are: (1) the realistic behavior of volcanic clouds can be represented efficiently; (2) the various shapes of volcanic clouds can be generated by changing only several parameters.

Received January 31, 2003; accepted July 4, 2003.
Communicated by Ruay-Shiung Chang.
* A preliminary version of this paper has been presented at 2002 International Computer Symposium, 2002.

2. RELATED WORK

Although there is almost no research on modeling of volcanic clouds in the field of computer graphics, there is much research on complex behavior of fluids such as smoke [5, 7, 21], clouds [1, 2, 9, 13, 18], water [4, 6, 8], flames [17, 20, 32], etc.

Kajiya and Herzen proposed a simulation method for cloud by solving the Navier-Stokes equations [13]. However, at that time (1984), they could do calculations on a small number of voxels due to the lack of the computational ability. Therefore their method can not represent a realistic cloud. Foster and Metaxas proposed a method that can generate realistic motion of turbulent smoke on relatively small number of voxels [7], but this method is stable only when the time step is very small and costs a lot of time for the calculation. Stam introduced the semi-Lagrangian advection scheme to calculate the advection term of the Navier-Stokes equations [21]. By using the semi-Lagrangian advection scheme, it is possible to calculate the advection term of the Navier-Stokes equations stably even if the time step is large. Fedkiw *et al*. provided a technique called vorticity confinement that is applied to Stam's model [5]. The vorticity confinement can represent small-scale vortexes lost during the numerical calculation process (refer [22] for details). However, the methods provided by Stam and Fedkiw *et al*. were premised on a relatively small space such as inside a room. Therefore their methods cannot take some factors which are related to height, like the variation of the atmospheric density with respect to height, into consideration. To overcome this difficulty, we introduced some parameters that are functions of height, although the fluid solver of our model is similar to the solver proposed by Fedkiw *et al*.

The CML method was originally developed by Kaneko and is an extended method of cellular automaton [15]. Yanagita and Kaneko proposed a method for the modeling and characterization of cloud dynamics using the CML method [31]. Miyazaki *et al*. extended their work and applied it to the modeling of clouds [18]. Their method can generate realistic clouds in a practical calculation time. However, the method provided by Miyazaki *et al*. is designed only to animate clouds, and cannot be applied to volcanic clouds directly.

In the field of earth and planetary sciences, there are many papers about the dynamics and modeling of volcanic clouds. One of the significant researches is proposed by Woods [25]. He analyzed the dynamics of the vertical structure of volcanic clouds, and proposed a one-dimensional model of volcanic clouds. Woods' model was followed by Dobran and Neri [3], Woods and Bower [26], and Neri and Macedonio [19]. However, since the models are one-dimensional, it is hard to apply them to three-dimensional simulations directly. Valentine and Wohletz proposed an axisymmetric two-dimensional model of volcanic clouds [24]. Moreover, their model was followed by Ishimine and Koyaguchi [11], Ishimine [12], and Susuki [23]. Their works made great progress since they made it possible to visualize the shapes of volcanic clouds. However, since their models are still axisymmetrically two-dimensional, they are also hardly to be directly applied to three-dimensional simulations. Although Suzuki suggested that the model proposed in [23] could be extended to be three-dimensional, the computational cost is too expensive.

3. BASIC IDEA

There are many factors that decide the shape of volcanic clouds. Among these factors, the eruption magnitude, buoyancy, decrease in volcanic cloud density, and temperatures of the magma and volcanic clouds are the most important. Since the ascending current due to the temperature of the magma can be considered to be the eruption velocity, and the buoyancy due to the temperature of the volcanic clouds can be considered to be the buoyancy generated by the difference between the volcanic cloud density and the atmospheric density. Hence, the temperatures of the magma and the volcanic clouds can be simplified to enhance simulation speed. Therefore, the proposed model is designed by taking the following important factors that decide the shape of the volcanic clouds.

- **Eruption magnitude:** The eruption magnitude is decided by the initial velocity and density of volcanic clouds, and it depends on the scale of the volcanic clouds.
- **Buoyancy:** The buoyancy is generated by the difference between the volcanic cloud density and the atmospheric density. The typical conically shaped clouds are generated due to the buoyancy.
- **Decreasing of the volcanic cloud density:** The volcanic cloud density can be decreased due to the loss of pyroclasts (fragments of magma). The diversities of the volcanic cloud shapes due to the differences of the contents inside the clouds are decided by the variety in the distribution of the loss of the pyroclasts.

Moreover, we make our model efficient and stable by using stable fluid solvers, which include the CML method that is a qualitative and efficient solver, and the semi-Lagrangian advection scheme that is a stable solver even if the time step is large.

4. MODEL

4.1 Evolution of Velocity Field

Since the atmospheric fluid and the volcanic clouds have small viscosity, and the eruption velocity of the volcanic clouds is less than the speed of sound, it is assumable that the following non-viscosity Naivier-Stokes equations can be used to describe the time evolution of the velocity field.

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla p + \mathbf{f}, \tag{2}$$

where **u** is a velocity vector, p is the pressure, and **f** is an external force that is applied to the velocity field. Eq. (1) means that the inflow and the outflow of a unit cell are balanced, and is called the “continuity equation”. This equation is a constraint which projects the velocity vector to the divergent free field. The first term of the right hand of Eq. (2) means the advection of the velocity vector, and is called the “advection term”. The second term means the variation of the velocity caused by the gradient of the pressure, and is called the “pressure term”. The third term means that the velocity is varied by the external force, and is called the “external force term”. In the proposed method, Eq. (1) and the pressure term of Eq. (2) are approximated by using the CML method. That is, the following approximated Navier-Stokes equation is obtained.

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \eta \nabla(\nabla \cdot \mathbf{u}) + \mathbf{f}\ , \tag{3}$$

where η is a positive constant which means the rate of diffusion, and is called the “diffusion coefficient”. Actually, the diffusion coefficient controls the scale of vortexes.

The approximated Navier-Stokes equation does not need iterative calculation to solve the continuity and the pressure effect, although iterative calculation is generally needed to solve the Poisson equation in other methods.

4.2 Evolution of Volcanic Clouds

Volcanic clouds are transported by the atmospheric fluid, and the volcanic cloud density is decreased due to the loss of the pyroclasts. Hence, the following equation for volcanic cloud density ρ can be defined.

$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho - \kappa(z)\rho\ , \tag{4}$$

where $\kappa(z)$ is called the “decreasing rate” and should be set with the following two considerations.

- Near the vent, the volcanic clouds include many large pyroclasts called the “volcanic blocks”. Therefore, ρ decreases rapidly due to the fall of the volcanic blocks. To simulate this phenomenon, $\kappa(z)$ needs to be set large in this region.
- In a higher region, the volcanic clouds consist of many small pyroclasts called “volcanic ash” and air, and then the pyroclasts are lost slowly. Therefore, $\kappa(z)$ needs to be set small in this region.

The diversity of the volcanic cloud shapes due to the differences of the constituents of the volcanic clouds can be represented by setting $\kappa(z)$.

4.3 Buoyancy

Buoyancy occurs due to the difference between the volcanic cloud density and the atmospheric density, and affects the velocity field. The buoyancy $\mathbf{f}_{buoy}$ is defined by Eq. (5). When the vertical component of $\mathbf{f}_{buoy}$ becomes negative, buoyancy works vertically downward. In this case, $\mathbf{f}_{buoy}$ works as gravity.

$$\mathbf{f}_{buoy} = \alpha(\rho_{atm}(z) - \rho)\mathbf{z}, \tag{5}$$

where α is a positive constant which controls the strength of the buoyancy, $\mathbf{z}$ is a vertically upward unit vector, and $\rho_{atm}(z)$ is the atmospheric density, which is defined as an exponential function of height as:

$$\rho_{atm}(z) = \rho_0 \exp\left(-\frac{z}{H_e}\right), \tag{6}$$

where ρ_0 is the atmospheric density at the sea level (z = 0), and H_e is the degree of variation of atmospheric density with respect to height, and called the "scale height".

Buoyancy plays an important role in deciding the shape of the volcanic clouds, and the following dynamics generate the conically shaped clouds.

- In the region where height is low is called the "gas thrust region", the atmospheric density is less than the volcanic cloud density. Thus, the perpendicular component of $\mathbf{f}_{buoy}$ becomes negative, and the buoyancy works perpendicularly downward. However, the momentum of the eruption is more dominant than buoyancy. Therefore, volcanic clouds are delivered upwards.
- In a higher region which is called the "convective region", the atmospheric density is larger than the volcanic cloud density. Thus, the perpendicular component of $\mathbf{f}_{buoy}$ becomes positive, and the buoyancy works perpendicularly upward. Hence, the volcanic clouds are delivered upwards.
- The region which is higher than the convective region is called the "umbrella region", where the atmospheric density and the volcanic cloud density are almost balanced. Therefore, $\mathbf{f}_{buoy}$ becomes almost $\mathbf{0}$, and the volcanic clouds are no longer delivered upwards.

The proposed approach can simulate realistic volcanic cloud behavior by satisfying these dynamics.

5. FLUID SOLVER

5.1 Setting of Analysis Space

The analysis space is represented as $n_x \times n_y \times n_z$ voxels. Each voxel is a cube with a uniform volume. The velocity vector $\mathbf{u}$ and the volcanic cloud density ρ are defined as the state variables at the center of each voxel. In the initial state, $\mathbf{u}$ is set to a small value randomly, and ρ is set to zero. However, for the voxels located in the mountain (the hatched squares in Fig. 1), $\mathbf{u}$ is set to a zero vector. Then, the decreasing rate $\kappa(z)$ and the strength of the side wind $\mathbf{f}_{wind}(z)$ can be defined by dragging the corresponding control points of the Bézier curves (the dots in Fig. 2) of a GUI shown in Fig. 2. The horizontal direction of $\mathbf{f}_{wind}(z)$ can be changed. The atmospheric density $\rho_{atm}(z)$ is defined as Eq. (6), and is shown in the GUI. The horizontal and vertical axes of the GUI are the standardized value of each function and height, respectively. In the GUI, curve-a denotes a decreasing rate, curve-b denotes the atmospheric density, and curve-c denotes the strength of the side wind. Finally, the eruption velocity $\mathbf{u}_{src}$ and the initial volcanic cloud density ρ_{src} that decide the eruption magnitude are assigned to the voxels corresponding to the vent (the circle in Fig. 1). It is also possible to make the volcanic clouds erupt with a spread. In our method, by changing only these parameters, to represent the diversity of the volcanic cloud shapes can be represented.

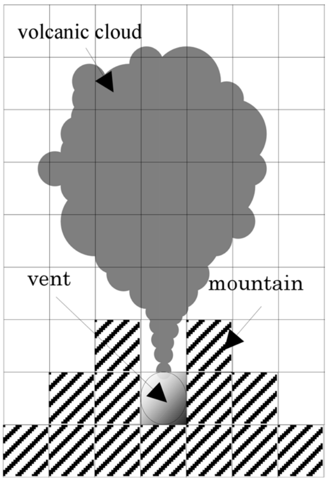


Fig. 1. Outline of the analysis space.

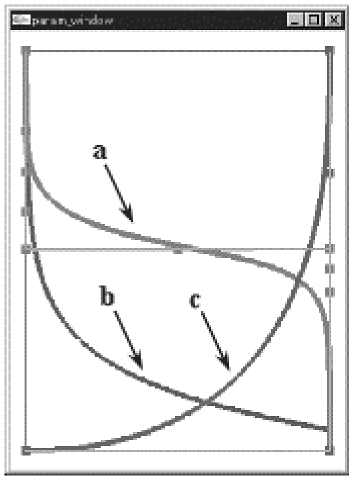


Fig. 2. The GUI for setting the functions of height.

5.2 Sequential Solver

The time evolution of the volcanic cloud behavior can be obtained by iterating as follows. In these processes the velocity vector **u** and the volcanic cloud density ρ are updated.

1. **Add force:** “Add force” is the process of adding an external force to the velocity fluid.
2. **Advect:** “Advect” is the process to advect the state variables.
3. **Pattern:** “Pattern” is the process of generating vortex patterns using the CML method.
4. **Decrease:** “Decrease” is the process of decreasing the volcanic cloud density with respect to the loss of the pyroclasts.

5.2.1 Add force

In the Add force process, the effect of the external force as the third term of the right hand side of Eq. (3) is calculated. In our method, the external force **f** is the sum of the buoyancy and the strength of the side wind $\mathbf{f}_{wind}(z)$. Hence, by assuming that **f** is unchangeable within a time step Δt, the equation for updating the velocity vector **u** is:

$$\mathbf{u}^*_1 = \mathbf{u} + (\mathbf{f}_{buoy} + \mathbf{f}_{wind}(z))\Delta t, \quad (7)$$

where $\mathbf{u}^*_1$ is the updated velocity vector in this process.

5.2.2 Advect

In the Advect process, the effects of the advection as the first term of the right hand of Eq. (3) and the first term of the right hand of Eq. (4) are calculated. For this calculation, we use the semi-Lagrangian advection scheme. As illustrated in Fig. 3, a path $\mathbf{p}(\mathbf{x}, s)$ is defined as a parametric function of time parameter s by tracing a particle, which is located in position $\mathbf{x}$ backwards along the velocity field at time t. Thus, $\mathbf{p}(\mathbf{x}, s)$ represents the position where the particle existed at time $(t - s)$.

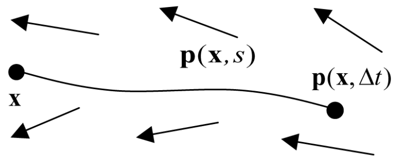


Fig. 3. Path setting in the semi-Lagrangian advection scheme.

The state variables at position $\mathbf{x}$ at time $(t + \Delta t)$ are advected from position $\mathbf{p}(\mathbf{x}, \Delta t)$. Hence, the equations for updating the state variables are:

$$\mathbf{u}^*_2(\mathbf{x}) = \mathbf{u}^*_1(\mathbf{p}(\mathbf{x}, \Delta t)) , \tag{8}$$

$$\rho^*_1(\mathbf{x}) = \rho(\mathbf{p}(\mathbf{x}, \Delta t)) , \tag{9}$$

where $\mathbf{u}^*_2$ and ρ^*_1 are the updated velocity vector and the updated volcanic cloud density in this process, respectively.

5.2.3 Pattern

In the Pattern process, the effect of the generating vortex patterns as the second term of the right hand of Eq. (3) is calculated. The CML method is used for this calculation. The equation for updating the velocity vector $\mathbf{u}^*_2$ is:

$$\mathbf{u}^* = \mathbf{u}^*_2 + \eta\nabla(\nabla \cdot \mathbf{u})\Delta t , \tag{10}$$

where $\mathbf{u}^*$ is the updated velocity vector in this process, and also the final updated velocity vector in the all sequential processes. A discrete version of Eq. (10) is shown in Eq. (11). Here, we only describe the equation for updating $u_{i,j,k}$, which is the u component of the velocity vector $\mathbf{u}(= (u, v, w))$ of voxel (i, j, k).

$$u^*_{i,j,k} = u^*_{i,j,k} + \eta\{(u_{i+1,j,k} + u_{i-1,j,k} - 2u_{i+1,j,k}) + (v_{i+1,j+1,k} - v_{i+1,j-1,k} - v_{i-1,j+1,k} + v_{i-1,j-1,k} + w_{i+1,j,k+1} - w_{i+1,j,k-1} - w_{i-1,j,k+1} + w_{i-1,j,k-1})/4\}\Delta t, \tag{11}$$

where $u^*_{i,j,k}$ is the u component of the velocity vector of the voxel (i, j, k) after being updated by the Pattern process. The v and w components can be updated similarly.

5.2.4 Decrease

In the Decrease process, the decrease in volcanic cloud density as the second term of the right hand of Eq. (4) is calculated. The equation for updating the volcanic cloud density ρ^*_1 is:

$$\rho^* = \rho^*_1 - \kappa(z)\rho\Delta t\ , \tag{12}$$

where ρ^* is the updated volcanic cloud density in this process, and also the final volcanic cloud density in the all processes.

$\mathbf{u}^*$ and ρ^* are the velocity vector and the volcanic cloud density after updated while a time step Δt, respectively.

6. RESULTS

The images generated by the proposed method are shown in Figs. 4-7. Figs. 4 and 5 show the volcanic clouds when there is no side wind. The relative profiles of the height function are also shown in Figs. 4-6. Fig. 4 shows the case when the decreasing rate in the gas thrust region is set to a relatively large, so that the volcanic clouds include many large pyroclasts. Conical volcanic clouds are generated as a result. Fig. 5 shows the case when the decreasing rate in the gas thrust region is set to be relatively small, hence the volcanic clouds consist of small pyroclasts and air. Spreading volcanic clouds are generated as a result. Fig. 6 shows a sequence of images of an animation of the volcanic clouds affected by side wind. The parameters except the strength of the side wind are set as in Fig. 4. The sharpness of the conic shape of the volcanic clouds depends on the decreasing coefficient in the gas thrust region (curves-a in Figs. 4-6), and the curved condition of the volcanic clouds can be controlled by adjusting the strength of the side wind (curve-c in Fig. 6).

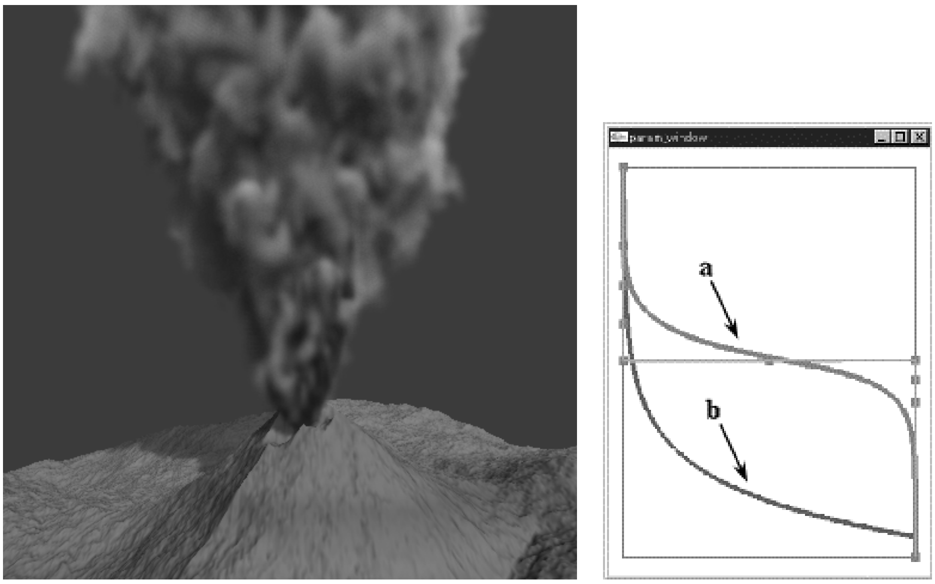


Fig. 4. Conical volcanic clouds generated by the proposed method. The diffusion coefficient (curve-a) when z is small was set relatively large.

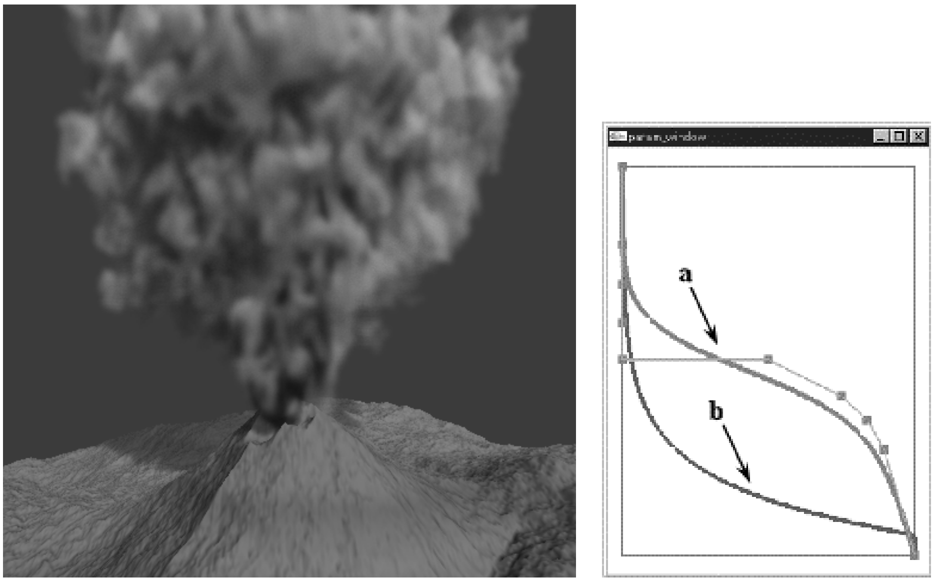


Fig. 5. Spreading volcanic clouds generated by the proposed method. The diffusion coefficient (curve-a) when *z* is small was set relatively small.

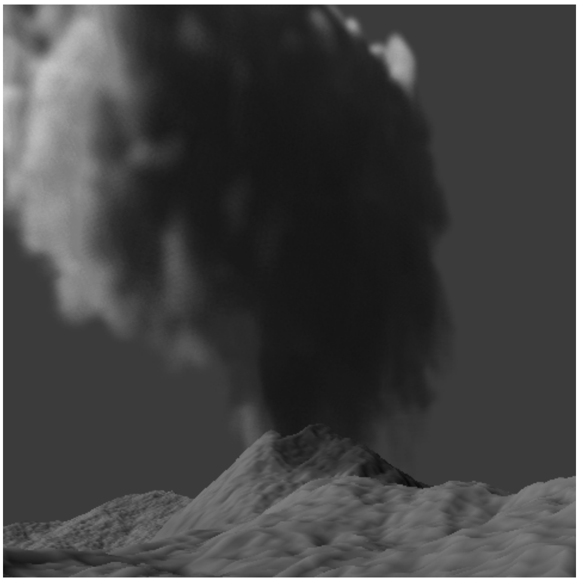
All of the images are visualized as the simulation results on $100 \times 100 \times 100$ voxels, by using the rendering method proposed by Dobashi *et al*. [14]. The computational time of the simulation was approximately 1 second per time step on an Intel Pentium 4 2.8 GHz CPU machine, and the computational time was almost proportion to the number of voxels. When the CML method is not used, the simulation time is several seconds at least depending on the Poisson equation solver.

Fig. 7 shows photographs of real volcanic clouds and the images of volcanic clouds generated by the proposed method for comparison. The shapes of clouds' outline of the photographs are similar to the images generated by the proposed method. This can show the representation ability of our model.

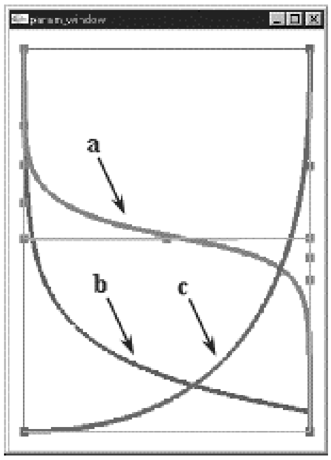
7. CONCLUSIONS

In this paper, a method for modeling volcanic clouds using the CML method is presented. The major features of our method are:

- The realistic behavior of volcanic clouds is represented by considering the eruption magnitude decided by the eruption velocity and initial volcanic cloud density, the buoyancy generated by the difference between the volcanic cloud density and the atmospheric density, and the decreasing of the volcanic cloud density due to the loss of the pyroclasts.
- Various shapes of volcanic clouds can be generated by changing only some parameters.
- An efficient simulation is achieved by using the CML method.



1500th frame



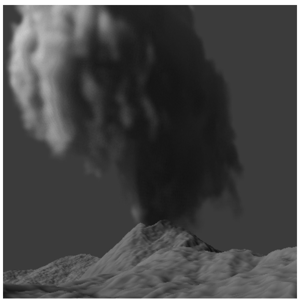
The side wind (curve-c) is added.

250th frame

500th frame

750th frame

1000th frame



1250th frame

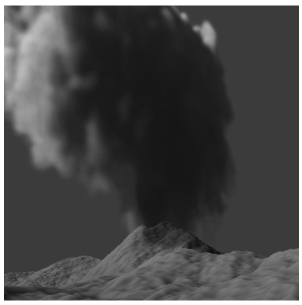


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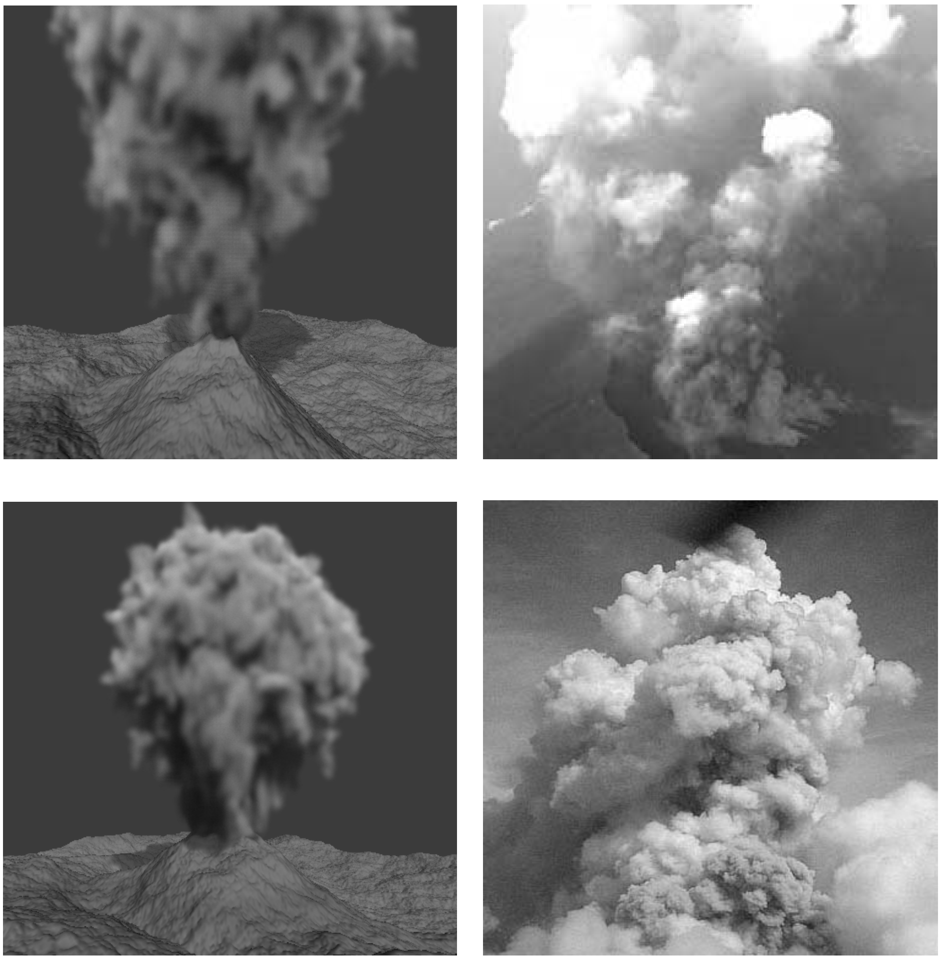
Fig. 6. Volcanic clouds affected by a side wind.



Images generated by the proposed method. Photographs.

Fig. 7. Comparison photographs of real volcanic clouds and volcanic clouds generated by the proposed method.

To enhance the reality of the images generated by our method, we should take account the expansion of the volcanic clouds due to the mixing of the pyrocalsts and the surrounding air into the model. Moreover, to reduce the calculation time of the simulation, it is a good idea to implement the proposed model by using graphics hardware. Harris *et al*. proposed a fast calculation method of CML using graphics hardware [10], although graphics hardware is generally used for rendering. The implementation of the CML part of the proposed model using graphics hardware will be a part of future work.

ACKNOWLEDGMENT

We thank Bing-Yu Chen (National Taiwan University) for many useful suggestions.

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