

# Twisting, Tearing and Flicking Effects in String Animations

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**Abstract.** String-like objects in our daily lives including shoelaces, threads and rubber cords exhibit interesting behaviors such as twisting, tearing and bouncing back when pulled and released. In this paper, we present a method that enables these behaviors in traditional string simulation methods that explicitly represent a string by particles and segments. We offer the following three contributions. First, we introduce a method for handling twisting effects with both uniform and non-uniform torsional rigidities. Second, we propose a method for estimating the tension acting in inextensible objects in order to reproduce tearing and flicking (bouncing back); whereas the tension for an *extensible* object can be easily computed via its stretched length, the length of an *inextensible* object is maintained constant in general, and thus we need a novel approach. Third, we introduce an optimized grid-based collision detection for an efficient computation of collisions. We demonstrate that our method allows visually plausible animations of string-like objects made of various materials and is a fast framework for interactive applications such as games.

## 1 Introduction

String-like deformable objects play an important role to represent hair strands, threads, elastic rods, cables and ropes in computer graphics. For realistic animations of such objects, we have to reproduce their interesting behaviors such as bending, stretching, twisting, tearing and flicking when pulled and released, according to their material properties. For example, threads are made of yarn that is barely stretched but easy to tear. An elastic rod made of rubber can be twisted and flicked but hard to break. A cable that has non-uniform density distribution within its cross-section or a partially braided rope such as fourragère has a non-uniform torsional rigidity. In the rest of this paper, we refer to such string-like objects as *strings*.

To simulate a string, several traditional methods can be used such as mass-spring systems [13,15], rigid multi-body serial chains [7], geometric approaches [12,

14] and elastic energy-based models [2, 3, 17]. However, all behaviors (twisting, tearing and flicking) of a string are not introduced together in a single framework by these methods.

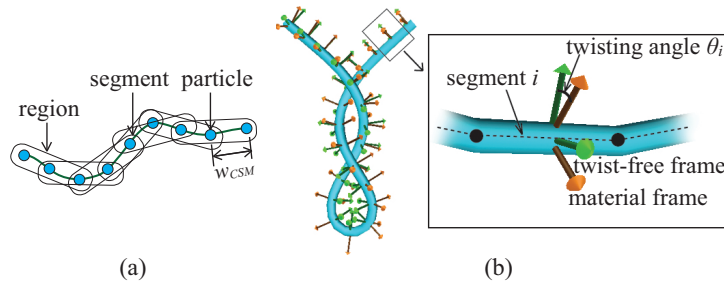
Handling of inextensible strings, such as threads and cables, poses another technical challenge. To prevent inextensible strings from excessive elongation, many length-constraint schemes called *strain limiting* have been developed. With strain limiting, however, the tearing simulation becomes difficult; whereas an *extensible* string will break when its length or strain reaches a certain breaking point, we cannot see when an *inextensible* string will tear based on the constrained length. Moreover, beside the fact that an inextensible string is not elongated by their own weight, under a large applied force such as a large pulling force, the string should be elongated according to its material property. However, the strain limiting causes the material property unrelated to the applied force.

In this paper, we present a method that can handle twisting, tearing and flicking of strings in real-time. Our method is a simple pseudo-physically-based model which is easy to implement, yet visually plausible results can still be achieved. Our method is applicable to traditional simulation methods that explicitly represent a string by particles and segments. Our implementation is based on *Chain Shape Matching* (CSM) [14], which is a simplified version of the more versatile deformation method, *Lattice Shape Matching* (LSM) [12], since CSM inherits and enhances several advantages of LSM (e.g. CSM is fast, easy to implement and numerically stable). Specifically, we offer the following three contributions:

1. We introduce a simple method for twisting effects by adding twisting angles into each segment in a string which can handle both uniform and non-uniform torsional rigidities (Sec. 4).
2. We propose a method for estimating the tension for tearing and flicking effects in an inextensible string whose actual stress and strain values are constrained from the strain limiting (Sec. 5).
3. We introduce a collision searching scheme for efficient collision handling using a grid-based data structure which has a less number of neighbors to be searched compared to typical searching schemes. (Sec. 6).

## 2 Related Work

**Simulation of twisting strings:** Many researches on the twisting effect in string simulation introduced various models for solving the Cosserat and Kirchhoff energy equations. Bertails *et al.* [3] introduced a mechanical model called *super helices* for simulating human hair based on the Kirchhoff theory. However, handling collision responses is not straightforward due to the implicit representation of hair strands. Spillmann and Teschner [17] explicitly represented the centerline of an elastic string and used the finite element method (FEM) to solve the Cosserat energy equation. Recently, Bergou *et al.* [2] introduced a discrete model for simulating elastic strings based on the Kirchhoff theory. However, the twisting angles are computed using quasi-static assumption, thus the twisting of non-uniform torsional rigidity along the string is not addressed. There are



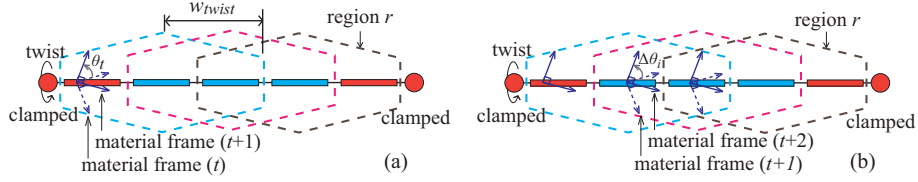
**Fig. 1.** Our string model. (a) Multiple overlapping chain region in CSM. (b) A twisting angle of a segment of an elastic string is an angle between a twist-free frame and a material frame.

also several works on pseudo-physical models that can capture the twisting effect without solving the energy equations. Hadap [7] introduced a model for capturing the torsion effect by integrating a torsion spring into each joint of rigid links. However, strings cannot be stretched and collision handling is not straightforward, because motion is propagated from top to bottom in one single pass (not affect backward). Selle *et al.* [15] represented a hair strand by a chain of tetrahedrons of springs and captured the torsion effect by introducing appropriate altitude springs. However, the configuration of springs is complex, auxiliary particles are required along a string.

**Strain limiting for inextensible strings:** In order to handle inextensible objects simulated by deformation models, a variety of methods for stretch resistance have been continuously proposed; from Provot’s iterative post-processing edge constraint [11], to a more recent constraint method based on impulse [5]. Some alternative ways of stabilizing stiff simulation were also proposed [1, 6, 10]. These methods, and many of their sequels, have a common goal to limit the maximal strain to a certain threshold. Accordingly, these kinds of methods are problematic in case of excessive stretch or when rupture should occur. Metaaphanon *et al.* [9] proposed a method to deal with cloth tearing using a mass-spring model. However, it tears cloth by checking lengths of springs; when and where yarns of cloth are cut were not directly related to user-applied external forces and cloth material properties, but dependent on how the method constrains the springs.

### 3 Chain Shape Matching (CSM)

Before describing the details of our algorithms, this section first briefly introduces *Chain Shape Matching* (CSM) [14], the basis model used in this paper. In CSM, a string is represented as a chain of particles connected by segments (see Fig. 1 (a)). The particles are grouped into multiple overlapping *chain regions* with the *region half-width*  $w_{CSM} \in \{1, 2, 3, \dots\}$ . The chain region half-width corresponds to the stiffness of the string. The particles are independently moved by external forces, and then an optimal rigid transformation (i.e., rotation and translation)



**Fig. 2.** (a) An elastic string clamped at both ends is twisted at one end with a twisting angle  $\theta_t$ . (b) The increment of the twisting angle is propagated to next segments.

of each region is computed. The rigidly-transformed positions of the particles are called *goal positions*. The goal position of each particle is weighed in the overlapping regions by *particle per-region mass*. Finally, each particle is updated toward the goal position.

## 4 Twisting Effects

Based on CSM, a string in our model is represented as a chain of  $(n+1)$  particles connected by  $n$  segments (Fig. 1 (a)). A segment  $i \in \{1, 2, \dots, n\}$  has a twisting angle  $\theta_i$  tracking how much the segment is twisted. The twisting angle can be represented as an angle between a *twist-free frame* (*bishop frame*) and a *material frame* (Fig. 1 (b)). In the initial state, we specify an initial angle  $\theta_i^0$  of each segment  $i$  according to the shape of the string. The twisting angle is assigned for each segment, not for each particle, to avoid the ambiguity.

The behavior of twisting can be clearly observed when a string clamped at both ends is twisted at one end. Therefore, we use this scenario for our explanation (Fig. 2). When we twist one clamped end with an angle  $\theta_t$ , the angle  $\theta_i$  of the segment is increased. The increment of the twisting angle of the segment is propagated to the next segments in order to minimize the elastic energy. We compute a goal twisting angle for each segment, similarly to finding a goal position for each particle in shape matching.

First, we group the segments into multiple overlapping chain regions with the region half-width  $w_{twist} \in \{1, 2, 3, \dots\}$  which affects the propagation speed of twisting angles in the string; the larger the  $w_{twist}$  is, the faster the change of twisting angles is propagated. The size of each region in a string can be varied for handling non-uniform torsional rigidity. The twisting angle increment  $\Delta\theta_k^{region}$  of each region  $k$  is computed by averaging the  $\Delta\theta_j = \theta_j - \theta_j^0$  weighted by mass  $m_j$  for the set of segments within the region  $S_k$ :

$$\Delta\theta_k^{region} = \frac{\sum_{j \in S_k} (\theta_j - \theta_j^0) m_j}{\sum_{j \in S_k} m_j}. \quad (1)$$

Then,  $\theta_i$  of each segment  $i$  is updated with the twisting angle increment  $\Delta\theta_i^{segment}$ . The goal twisting angle increment  $\Delta\theta_i^{segment}$  is calculated by summing the

$\Delta\theta_k^{region}$  of each region  $k$  that segment  $i$  belongs to:

$$\Delta\theta_i^{segment} = \sum_{k \in \mathfrak{R}} \Delta\theta_k^{region}, \quad (2)$$

$$\theta_i \leftarrow \theta_i + \Delta\theta_i^{segment}, \quad (3)$$

where  $\mathfrak{R}$  is a set of regions that segment  $i$  belongs to. The twisting force  $\mathbf{f}_i^{twist}$  can be treated as an external force to particle  $i$  and derived from elastic energy equation [2] as follows:

$$\mathbf{f}_i^{twist} = \frac{\beta}{L} (\theta_{i+1} - 2\theta_i + \theta_{i-1}) \left( \frac{-\kappa \mathbf{b}_{i+1} - \kappa \mathbf{b}_{i-1}}{2l} \right), \quad (4)$$

$$\kappa \mathbf{b}_i = 2 \frac{\mathbf{e}_{i-1} \times \mathbf{e}_i}{|\mathbf{e}_{i-1}| |\mathbf{e}_i| + \mathbf{e}_{i-1} \cdot \mathbf{e}_i}, \quad (5)$$

where  $\kappa \mathbf{b}_i$  is the curvature binormal,  $\mathbf{e}_i$  is the segment vector,  $l$  is the length of the segment,  $\beta$  is the twisting stiffness of the string and  $L$  is the total length of the string.

## 5 Tearing and Flicking Effects

This section briefly reviews the material science of a string, and then describes *strain limiting*, a technique to constrain the length of an inextensible object. Finally, we describe the way to estimate the tension in an inextensible string in order to handle tearing and flicking.

### 5.1 Stress and Strain

In material science, the strength and elongation of a string is associated with its stress-strain curve [4]. The stress  $\sigma$  of a string is the average force per unit area of a cross-section surface:

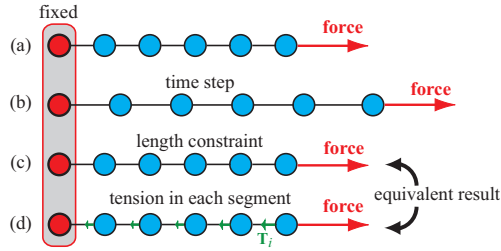
$$\sigma = \frac{\|\mathbf{F}_n\|}{A}, \quad (6)$$

where  $A$  is the cross-sectional area and  $\mathbf{F}_n$  is the normal force. The total force acts on the cross-section surface in the surface normal direction. The strain  $\varepsilon$  of a string is expressed as the ratio of the elongation  $\Delta L$  to the initial length  $L_0$  of the string:

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}, \quad (7)$$

where  $L$  is the current length of the string.

Along the curve, the material exhibits elastic behaviors until the *yield point*; prior to the yield point the material will return to its original shape if the applied force is removed. The slope of this elastic region is the *Young's modulus*  $E = \sigma/\varepsilon$ .



**Fig. 3.** A simple example of the tension computation. From (a) to (c), an ordinary length-constrained string simulation is performed. The tensions make the particles move from their unconstrained positions to the constrained positions in (d), yielding an equivalent result for (c).

Once the yield point is passed, the material becomes plastic; some fractions of the deformation will be permanent and non-reversible. As deformation continues, the material will break when the stress or strain reaches the *rupture point*.

The stress-strain curve can be derived via a strength testing of a material sample stored as a data set of the experimental result. However, the stress-strain curve of most materials in the elasticity state is linear with the Young's modulus as its slope. Therefore, the part of the curve from the origin to the yield point can be stored as a constant value. Still, the data set is required for the curve in the plasticity state. In our implementation, we simply approximate the curve by a line with a constant slope that fits the curve best. As a result, our implementation uses two constant values to represent the stress-strain curve in elasticity and plasticity states together with two constants for yield point and rupture point.

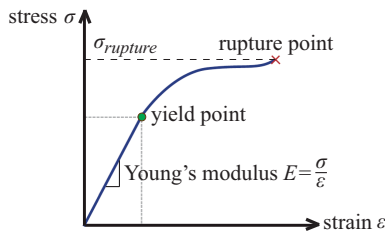
## 5.2 Strain Limiting

To prevent segments from stretching excessively, position constraints are imposed so that the length of each segment  $i$  does not exceed a certain threshold  $L_i^{max}$ . Since correcting the length of one segment may change the length of other segments, iterative adjustment is required. Each constraint is solved independently one after the other as done in [10].

## 5.3 Tension Estimation

As previously stated, due to the constraint on lengths unrelated to applied forces, actual stress and strain values cannot be directly computed from the simulation result. Here we propose a novel approach to estimate the actual stress and strain values for inextensible strings.

The actual stress and strain values can be computed by estimating the *tensions* in the string. To derive the tensions, we also consider the particle positions computed *without* strain limiting. We model the *tension*  $\mathbf{T}_i$  of a segment  $i$  as



**Fig. 4.** Typical stress-strain curve of a string.

a stiff force that makes its particles  $i$  and  $i + 1$  at both ends move from their unconstrained positions  $\mathbf{x}'_i$  and  $\mathbf{x}'_{i+1}$  (Fig. 3 (b)) to the constrained positions  $\mathbf{x}_i$  and  $\mathbf{x}_{i+1}$  (Fig. 3 (c)). In our implementation, we compute the tension as follows:

$$\mathbf{T}_i = k_{stiff}(\|\mathbf{x}'_{i+1} - \mathbf{x}'_i\| - \|\mathbf{x}_{i+1} - \mathbf{x}_i\|)\mathbf{t}_i, \quad (8)$$

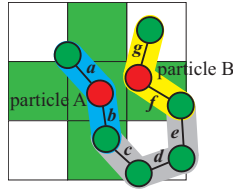
where  $k_{stiff}$  is a coefficient and  $\mathbf{t}_i$  is a unit vector from particle  $i$  to  $i + 1$ . The tension derived this way is used to reproduce tearing and flicking as well as plastic behaviors of a string, as described in Sec. 5.4.

#### 5.4 Tearing and Flicking a String

For tearing computation, we assign a rupture point or a stress threshold  $\sigma_{rupture}$  for each segment. If segment's stress exceeds its stress threshold, the segment will be broken. The applied stress  $\sigma_i$  can be computed from tension  $\mathbf{T}_i$  in each segment using Eq. (6) with  $\mathbf{F}_n = \mathbf{T}_i$ .

Similarly, we can handle the behavior of flicking using the tension. When an inextensible string is pulled and released or torn apart, the applied stress is vanished but the strain of the segment from the elongated length still remains. The bouncing back force can be computed from an internal stress translated from the strain by referencing the stress-strain curve (Fig. 4). However, the elongated length is computed from the tension, i.e., we can directly use the tension as the bouncing back force. Note that, without this technique, the string will just fall down quietly by the gravity force because the elongated length is very small.

As can be seen in the stress-strain curve (Fig. 4), a real string is lengthened according to the stress in the elasticity and plasticity states prior to the rupture point. Therefore, the maximum length  $L_i^{max}$  of each segment used in strain limiting (Sec. 5.2) should be updated accordingly (otherwise the string does not elongate). For this, we look up strain  $\varepsilon_i$  corresponding to applied stress  $\sigma_i$  from the stress-strain curve, and use it to compute the appropriate value of  $L_i^{max}$  using Eq. (7),  $L_i^{max} = \varepsilon_i L_0 + L_0$ . In the elasticity state, the strain  $\varepsilon_i$  of a string becomes zero when applied forces are removed. However, when its strain exceeds the yield point (plasticity state),  $\varepsilon_i$  is still the same as the last time the forces are applied. Our method also modifies the radius of a segment in order to preserve the volume of the segment when stretched.



**Fig. 5.** A 2D example of our optimized searching scheme. When doing a collision detection between particles A and B, segment collision tests between  $ag$ ,  $af$ ,  $bg$  and  $bf$  capsules are tested.

## 6 Collision Handling

In this section, we introduce an optimized searching scheme for collision detection of strings. Previous works often use techniques based on bounding volume hierarchy (BVH) [15, 16, 18] for collision detection of strings. Apart from BVH, space partitioning using a grid-based data structure is a simple and efficient technique for collision detection of strings which have a large number of self-collisions. Specifically, we treat the segments as capsules and search for capsule collision pairs. For neighbor searches, we use a uniform grid of voxels. The number of voxels to be searched is 27 ( $= 3 \times 3 \times 3$ ) in a naïve approach. For better performance, we found that it suffices to search for colliding segments in only seven neighboring voxels (top, bottom, left, right, front, back and center voxels) under the following three specifications.

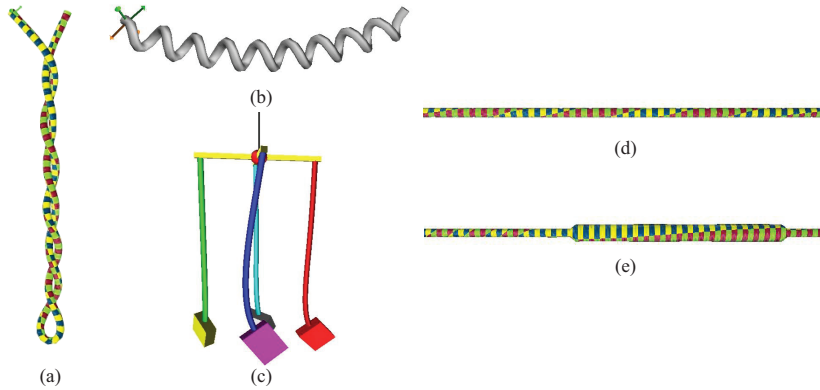
- Specifying the voxel size equal to or larger than segment length  $l$
- Storing indices of particles in each voxel
- Searching for capsule collision pairs from two adjacent segments of each particle in the seven neighboring voxels

For a better understanding, we describe using an example in 2D (five neighboring cells). The idea can be generalized to the 3D case in a straightforward manner. In Fig. 5, particles A and B are neighbors. Our method does the segment collision test between their two adjacent segments, i.e., pairs of segments  $ag$ ,  $af$ ,  $bg$  and  $bf$ . If two segments have an intersection, there is definitely a pair of particles at their both ends in the seven neighboring voxels that the intersection can be searched, even the intersection of segments is outside the seven neighboring cells. This could be easily proved, if one writes all possible cases in 2D with five neighboring cells (center, up, down, left and right).

The closest points of a pair of colliding segments  $i$  and  $j$  are indicated by fractions  $s \in [0, 1]$  and  $t \in [0, 1]$ , respectively. In order to move the colliding segments to the non-intersection positions, we compute a displacement vector between the closest points. Then, we move the both-end particles of each segment corresponding to the fractions  $s$  and  $t$  similar to [18].

Moving a colliding segment may make the string discontinuous, and thus we repeat shape matching until particle positions converge. Conversely, shape





**Fig. 6.** Our simulation results of twisting effects. (a) The twisting effect of a string clamped at both ends. The string is gradually twisted on the left end and finally twisted to form a loop. (b) A twisted string that forms a spiral shape like a telephone cord. (c) An application for a hanging mobile in wind forces. (d) A string with uniform torsional rigidity. (e) A string with non-uniform torsional rigidity.

matching may cause a collision again. As a result, iterations are required for both shape matching and collision constraints. To lessen the iterations, we temporarily make the masses of colliding particles heavier so that shape matching barely moves the particles. In our experiments, by making the colliding particles three times heavier, only one iteration suffices.

## 7 Results

Our implementation was written in C++ with OpenGL. All experiments were conducted on a desktop PC with an Intel Core i7 3.20GHz CPU and 6GB RAM.

Fig. 6 shows the results of our twisting simulation. The twisting of strings can reproduce phenomena such as an instability of bending and twisting called *buckling* which makes a string to form a spiral shape (Fig. 6 (a) and (b)). An application for a hanging mobile is also presented. In Fig. 6 (c), objects at the tips of strings are rotated by wind forces and the strings are twisted. With twisting effects, the strings twist back to the rest state, making the objects rolling back and forth. Fig. 6d and Fig. 6e show the twisting of strings with uniform and non-uniform torsional rigidities, respectively. The twisting angles in the string with non-uniform torsional rigidity are distributed more equally in the larger cross-section. Fig. 7 shows the tearing simulation results with the variation of rupture thresholds. The rupture threshold is assigned to all segments in the string. However, that kind of completely uniform strength is impossible in real string. Our method randomly alters the rupture threshold in each segment with the range of variation up to 0.01%. Please see the supplemental video for more variation tests. Animation sequences of flicking are shown in Fig. 8. Without

flicking, the string in Fig. 8 (a) falls naturally when an applied force is removed. The string in Fig. 8 (b) bounces back by the tensions when the applied force is removed. When the twisted string in Fig. 8 (c) is pulled and released, the twisting effect also occurs. Fig. 9 shows simulation results of a destruction of a hanging bridge. Wooden boards (rigid bodies) are tied with strings (ropes in this case) to build the bridge. The ropes are gradually torn apart from collisions of the wooden boards and incoming crates which cause high tensions in the ropes. We used particle-based simulation method [8] for rigid body simulation in our implementation. The video of our results can be found in the following link: <http://nis-lab.is.s.u-tokyo.ac.jp/~witawat/rod/stringAnimation.mov>

The breakdown computational time in each process for strings with a different numbers of particles is shown in Table 1. The strings in Fig. 6 (a), (b) and (c) consist of 100, 100 and 200 segments respectively, while each string in Fig. 7 and 8 has 150 segments. The number of segments in Fig. 9 is 746 segments. The computational time of the result in Fig. 9 is measured excluding the time for rigid body simulation.

**Limitations :** Our method has some limitations. As previously mentioned, our method is not a full physically-based model, thus, more advance physics behaviors such as spring-twisting pendulum and anisotropic bending in [2] are hard to generate. The rapid motion of strings could cause the strings to pass through each other or themselves. However, the problem did not occur in our experiments. In case of rapid motion, continuous collision detection should be considered.

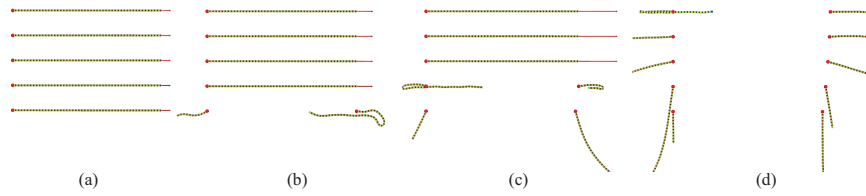
## 8 Conclusion and Future Work

We have introduced a simple model for simulating twisting, tearing and flicking of strings, which is fast, easy to implement and applicable to traditional simulation models. We have demonstrated that our method can handle twisting effects of strings with both uniform and non-uniform torsional rigidities. Using our method, the tension in an inextensible string can be estimated for generating tearing and flicking effects. A variation in the quality of strings can be achieved. Whilst our method is not physically-based, it can successfully reproduce the interesting behaviors of strings that would greatly enrich the realism of interactive applications such as games.

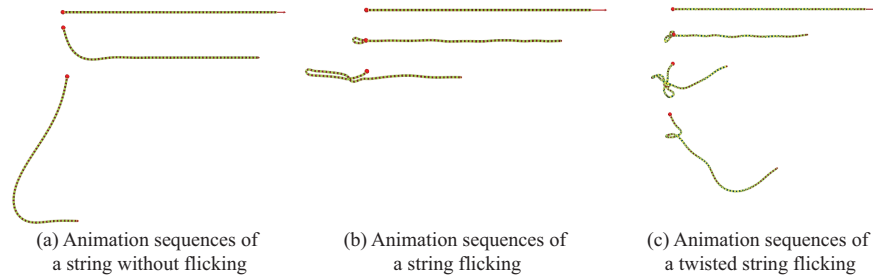
The collision between segments is treated as a collision between rigid segments. We would like to improve the collision detection algorithm to handle the collisions between deformable segments. We also would like to improve an overall performance with a GPU implementation.

## 9 Acknowledgement

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**Fig. 7.** Tearing simulation results of five strings with rupture thresholds. Rupture thresholds of strings are varying, increasing from bottom to top. The rupture threshold in each segment of the string is randomly altered with the range of variation up to 0.01%. As expected, the bottommost string, which had the lowest rupture threshold, was torn first at the weakest point as animation sequences from (a) to (d).



**Fig. 8.** Flicking animation sequences of strings from top to bottom.

**Table 1.** The computational time in milliseconds of each process in one time step. The time step in our implementation is 0.01 second.

| No. of segments | Time integration | CSM (Sec. 3) | Twisting comp. (Sec. 4) | Tension est. (Sec. 5) | Collision handling (Sec. 6) | Total time |
|-----------------|------------------|--------------|-------------------------|-----------------------|-----------------------------|------------|
| 100             | 0.011            | 0.086        | 0.098                   | 0.221                 | 1.31                        | 1.73       |
| 150             | 0.022            | 0.168        | 0.184                   | 0.424                 | 1.36                        | 2.16       |
| 200             | 0.029            | 0.221        | 0.237                   | 0.564                 | 1.37                        | 2.42       |
| 746             | 0.128            | 0.501        | 0.739                   | 1.67                  | 3.13                        | 6.17       |

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**Fig. 9.** Animation sequences of a hanging bridge colliding with incoming crates.

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